Simplified description of optical forces acting on a nanoparticle in the Gaussian standing wave

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We study the axial force acting on dielectric spherical particles smaller than the trapping wavelength that are placed in the Gaussian standing wave. We derive analytical formulas for immersed particles with relative refractive indices close to unity and compare them with the numerical results obtained by generalized Lorenz–Mie theory (GLMT). We show that the axial optical force depends periodically on the particle size and that the equilibrium position of the particle alternates between the standing-wave antinodes and nodes. For certain particle sizes, gradient forces from the neighboring antinodes cancel each other and disable particle confinement. Using the GLMT we compare maximum axial trapping forces provided by the Gaussian standing-wave trap (SWT) and single-beam trap (SBT) as a function of particle size, refractive index, and beam waist size. We show that the SWT produces axial forces at least ten times stronger and permits particle confinement in a wider range of refractive indices and beam waists compared with those of the SBT. © 2002 Optical Society of America

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1. INTRODUCTION

Within the past two decades optical trapping has proved to be an invaluable method for noncontact manipulation of nano-objects and micro-objects measurement of extremely weak forces and study of single molecule properties and surface properties. The most frequently used trapping set-up is based on a single laser beam tightly focused by an immersion microscope objective of high numerical aperture. This classical single-beam trap (SBT) set-up has been gradually modified by using interference of co-propagating laser beams, self-aligned dual beams, or optical fibers. Several laser beams, diffractive optics, and time sharing of a single beam were employed to create several optical traps. The common characteristic of all the above methods is that the axial force exerted on the object is smaller than the radial one. Recently it has been shown that the optical trapping can also be achieved in the standing wave created by interference of two counter-propagating coherent beams. In this case the axial force is stronger than the radial one because of strongly inhomogeneous optical intensity distribution in the periodic structure of the standing-wave nodes (minimums) and antinodes (maximums). Therefore particle confinement is achieved even in weakly focused or aberrated beams. An interesting problem that arises here is to study the properties of this type of optical trap [standing-wave trap (SWT)] and compare them with the classical SBT for different parameters of the trapped object (size, refractive index) and trapping beam (waist size).

Theoretical description of optical forces is simple only for very small particles—so-called Rayleigh particles—whose radius fulfills \( a \ll \lambda/20 \), where \( \lambda \) is the trapping wavelength in the medium. Such a small particle behaves as an induced elementary dipole, and the optical forces acting on it can be divided into two components, gradient and scattering forces. The gradient force comes from electrostatic interaction of a particle (dielectric) with an inhomogeneous electric field, and the scattering force results from the scattering of the incident beam by the object.

For particles larger than \( \lambda/20 \), a more complex concept involving the Maxwell stress tensor of the electromagnetic field surrounding the particle must generally be used. This requires knowledge of the total field outside the confined particle. The original theory based on plane-wave scattering has been gradually modified so that it can be applied to the case of a spherical or spheroidal particle placed into an arbitrary field distribution. It is commonly referred to as the generalized Lorenz–Mie theory (GLMT).

A recently presented intuitive approach assumes that the electric field creates such a large gradient force that the contribution of the scattering force to the total optical force is negligible. Consequently, only the gradient force controls the behavior of an irradiated dielectric object. The stress tensor contains only the terms depending on the electric field, and the problem is then reduced to the evaluation of the change in electrostatic energy after insertion of the dielectric into the inhomogeneous electrostatic field. Because the dielectric is assumed to be weak, its placement into the field does not change the ini-
tial field distribution appreciably. This allows use of an analytic expression for the optical force acting on spherical particles in an isotropic Gaussian field or on a cube in axially symmetric fields without further restriction on the particle size. This method will be called electrostatic approximation (ESA) in this paper.

The application of this approach is limited to cases where the scattering force can be neglected. It has been shown\textsuperscript{22} that if the Gaussian standing wave (GSW) is used for the optical trapping, the scattering force acting on the Rayleigh particle is proportional to the difference in the energy fluxes of counterpropagating waves. If the two waves have the same intensity, the scattering force is zero. A similar trend can also be expected for larger particles. Thus the ESA framework is well suited to the evaluation of the optical forces in the GSW.

2. ELECTROSTATIC APPROXIMATION APPLIED TO A SPHERICAL DIELECTRIC PARTICLE PLACED IN THE GAUSSIAN STANDING WAVE

A. General Description of the Optical Forces by Using Electrostatic Approximation

Let us assume that a dielectric object is placed into an inhomogeneous electromagnetic field in a dielectric liquid. If the sources of the field are kept fixed, the change in total energy of the field due to the insertion of the object can be written as the difference between the energy of the field before and after insertion of the object, respectively. The integration is over the whole space occupied by the field. During observation of the object we cannot follow the rapid optical frequencies; therefore, only the time-averaged values are accessible. It can be shown (see Appendix A) that if the object and the liquid are nonmagnetic, the time-averaged change in the field energy can be rewritten as

$$\Delta W = \frac{1}{2} \int_V \left( \mathbf{E} \cdot \mathbf{D} + \mathbf{H} \cdot \mathbf{B} \right) dV,$$

where \( \mathbf{E} (\mathbf{D}) \) and \( \mathbf{E}_0 (\mathbf{D}_0) \) are the electric field vector (electric displacement) and \( \mathbf{H} (\mathbf{B}) \) and \( \mathbf{H}_0 (\mathbf{B}_0) \) are the magnetic field vector (magnetic induction) before and after insertion of the object, respectively. The integration is over the whole space occupied by the field. During observation of the object we cannot follow the rapid optical frequencies; therefore, only the time-averaged values are accessible. It can be shown (see Appendix A) that if the object and the liquid are nonmagnetic, the time-averaged change in the field energy can be rewritten as

$$\langle \Delta W(\mathbf{r}, t) \rangle_T = -\frac{1}{2} \varepsilon_2 \alpha \int_{V_1} \langle \mathbf{E}(\mathbf{r}, t) \mathbf{E}(\mathbf{r}, t) \rangle_T dV,$$

where \( \alpha = \epsilon_1/\epsilon_2 - 1 \), \( \epsilon_i = n_i^2 \epsilon_0 \) is the permittivity of the particle \((i = 1)\), the liquid \((i = 2)\), and the vacuum \((i = 0)\). Symbols \( n_1 \) and \( n_2 \) represent the refractive indices of the particle and the liquid, respectively. The integration is now performed over the volume \( V_1 \) of the particle. If the particle is not strongly polarized, i.e., \( n_1 = n_2 \), the perturbed electric field \( \mathbf{E} \) can be approximated by the unperturbed one \( \mathbf{E}_0 \) and we obtain

$$\langle \Delta W(\mathbf{r}, t) \rangle_T = -\frac{1}{2} \varepsilon_2 \alpha \int_{V_1} \langle |\mathbf{E}(\mathbf{r}, t)|^2 \rangle_T dV.$$

Assuming that the incident beam is polarized mainly in the transverse direction (paraxial beam), we obtain, using the relation between the optical intensity and the field vectors,

$$\mathbf{I}_0(\mathbf{r}) = \langle S(\mathbf{r}, t) \rangle_T$$

$$= \langle \mathbf{E}_0(\mathbf{r}, t) \times \mathbf{H}_0(\mathbf{r}, t) \rangle_T,$$

$$= z_0 \langle n_2 \epsilon_0 |\mathbf{E}_0(\mathbf{r}, t)|^2 \rangle_T$$

$$= z_0 n_2 \epsilon_0 |\mathbf{E}_0(\mathbf{r})|^2/2 = z_0 I_0(\mathbf{r}).$$

Here \( z_0 \) is the unit vector in the direction of the light propagation and \( c \) is the speed of light. The equation for the energy change consequently takes the final form

$$\Delta W(\mathbf{r}) = \langle \Delta W(\mathbf{r}, t) \rangle_T = -\frac{\alpha n_2}{2 c} \int_{V_1} I_0(\mathbf{r}) dV.$$

The time-averaged force can be expressed by the divergence theorem

$$\mathbf{F}(\mathbf{r}) = -\nabla \langle \Delta W(\mathbf{r}, t) \rangle_T$$

$$= \frac{\alpha n_2}{2 c} \int_{V_1} \nabla I_0(\mathbf{r}) dV = \frac{\alpha n_2}{2 c} \int_{S_1} \mathbf{n} I_0(\mathbf{r}) dS,$$

where \( \mathbf{n} \) is the outward unit normal to the surface element \( dS \).

B. Analytical Formulas for a Spherical Dielectric Particle Placed in the Gaussian Standing Wave

Classical optical trapping requires a focused laser beam with a spot diameter comparable to the trapping wavelength. To achieve this, high-quality immersion objectives are usually used. In this case the beam goes through many dielectric interfaces (optical elements inside the objective, immersion oil layer, coverslip, and water layer), and the actual field distribution can differ considerably from the theoretical model, usually represented by a Gaussian beam.\textsuperscript{30} However, since we want to present here only the basic properties of the SWT, we

Fig. 1. Orientation of axes in the standing-wave apparatus. The \( z \) axis follows the direction of the reflected wave. Positive \( z_0 \) means that the beam waist of radius \( w_0 \) is located in the reflected wave.
have chosen a moderately focused Gaussian beam to reach a compromise among the exact description, speed of calculation, and availability of analytical results.

Let us assume that the GSW is created by interference of an incident wave and a wave reflected at a dielectric surface (see Fig. 1), and let us omit any multiple scattering between the object and the surface. Let the orientation of the z axis be parallel to the direction of the reflected beam. Therefore the “trapping force,” which acts against the incident wave, has positive sign in this coordinate system. The incident and the reflected Gaussian beams can be written as

\[ E_i(r_B, z_B) = E_{0i} \frac{w_0}{w_i} \exp\left(-\frac{r_B^2}{w_i^2}\right) \exp\left[i k(z_B + z_0)\right] \]

\[ E_r(r_B, z_B) = E_{0r} \frac{w_0}{w_r} \exp\left(-\frac{r_B^2}{w_r^2}\right) \exp\left[-i k(z_B - z_0)\right] \]

where \( E_{0i} \) is the electric field amplitude at the beam waist position, \( k \) is the wave number in the medium, \( \rho \) is the reflectivity of the surface, \( \psi \) is the phase shift after reflection [the Fresnel reflection coefficient has the form \( r_m = \rho \exp(-i\psi) \)], and \( z_0 \) is the distance of the beam waist from the surface (mirror). Positive (negative) \( z_0 \) corresponds to beam waist created in the reflected (incident) wave (see Fig. 1). Symbol \( w_0 \) is the beam waist size and \( w_i \) and \( w_r \) are the widths of the incident and the reflected beams at a given \( z_B \), respectively. \( R_i \) and \( R_r \) are the radii of the wave-front curvature for the incident and the reflected waves, respectively,

\[ w_{i0}(z_B) = w_0 \left[ 1 + \frac{(z_B + z_0)^2}{z_B^2} \right]^{1/2}, \]

\[ R_{i0}(z_B) = \frac{w_0^2}{z_B} \left[ 1 + \frac{z_B^2}{(z_B + z_0)^2} \right], \]

and \( z_R = kw_0^2/2 \) is the Rayleigh length. Under these assumptions we can write for the total intensity distribution

\[ I_{GSW}(r_B, z_B) = \frac{n_2 e_0 c}{2} |E_i(r_B, z_B) + E_r(r_B, z_B)|^2 \]

\[ = I_{00} \frac{w_0^2}{w^2} \exp\left[-(2r_B^2/w^2)\right] \]

\[ + \frac{\rho}{2} I_{00} \frac{w_0^2}{w^2} \exp\left[-(2r_B^2/w^2)\right] \]

\[ \times \exp\left[-(2r_B^2/w^2)\right] \cos \phi_{GSW} \]

\[ + \frac{\rho^2}{2} I_{00} \frac{w_0^2}{w^2} \exp\left[-(2r_B^2/w^2)\right], \]

Here, \( I_{00} \) is the on-axis intensity at the position of the beam waist and is related to the total power of the incident Gaussian beam by \( I_{00} = 2P/\pi w_0^2 \).

Because we assume that the dielectric object has spherical shape, it is useful to rewrite radial and axial cylindrical coordinates \((r_B, z_B)\) referenced to the beam by using spherical coordinates \((r, \chi, \phi)\) referenced to the sphere center, which is shifted from the center of the cylindrical coordinate system by \( r_s \) laterally and \( z_s \) axially: \( r_B^2 = r^2 \sin^2 \chi + r_s^2 + 2r r_s \sin \chi \cos \phi \), \( z_B = z_s + r \cos \chi \). In this case the final forms of the energy change [Eq. (5)] and the axial force [Eq. (6)] are

\[ \Delta W(r_s, z_s) = -\frac{\alpha n_2}{c} - 2\pi \int_0^a \int_0^\pi I_{GSW}(r_B, z_B) r_s^2 \sin \chi \cos \chi \sin \chi \cos \chi \cos \chi \]

\[ F_z(r_s, z_s) = \frac{\alpha n_2}{c} - 2\pi \int_0^a \int_0^\pi I_{GSW}(r_B, z_B) r_s^2 \sin \chi \cos \chi \cos \chi \]

These integrals can be expressed analytically only if one of the following approximations is employed.

1. \( a \ll w_0, a \ll \lambda \) (Rayleigh Particle)

The particle is so small that the intensity can be considered constant over the integration volume, and therefore it can be taken out of the integral in Eqs. (11) and (12). Consequently, we obtain

\[ \Delta W(r_s, z_s) = -\frac{2 n_2}{3 c} \frac{\alpha \pi a^3 I_{GSW}(r_s, z_s)}, \]

\[ F_z(r_s, z_s) = \frac{2 n_2}{3 c} \frac{\alpha \pi a^3 I_{GSW}(r_s, z_s)}, \]

Equation (14) is identical to the Harada et al. result for the gradient force in a paraxial beam if we use the substitution

\[ \alpha = m^2 - 1 = 3[(m^2 - 1)/(m^2 + 2)], \]

which, however, is correct only for \( m = 1 \). The difference between the left-hand and right-hand sides of Eq. (15) is 7% and 14.7% for \( m = 1.1 \) and \( m = 1.2 \), respectively.

Therefore the validity of ESA within the Rayleigh approximation can be extended into a region of higher refractive index by substituting Eq. (15) into Eq. (14).
2. \( a \ll w_0, a = \lambda \) (Intermediate Particle in Moderately Focused Beams)

This assumption means that over the integration volume the intensity does not change materially in the transverse direction, and therefore purely transverse-coordinate-dependent terms can be taken out of the integral in Eqs. (11) and (12). As the particle size is comparable to the wavelength, there is an intensity change in the axial direction that cannot be neglected over the integration volume. If all the terms connected with Eq. (9) are transformed into the spherical coordinates (see Appendix B) and the assumption \( a \ll w_0 \) is applied, we obtain the following expressions:

\[
\Delta W(r_s, z_s) = -\alpha \frac{n_2}{c} \left[ \frac{1}{w_i^2} \exp[-(2r_s^2/w_i^2)] + \rho^2 \frac{1}{w_r} \exp[-(2r_s^2/w_r^2)] \right] \left[ \frac{4}{3} a^3 + \frac{\rho}{k^2 w_i w_r} \exp[-r_s^2(1/w_i^2 + 1/w_r^2)] \right] \times (\sin 2ka - 2ka \cos 2ka) \cos \phi_s, \]

(16)

\[
F_z(r_s, z_s) = -\frac{n_2 \rho}{c} \frac{P}{k^2 w_i w_r} \rho \exp[-r_s^2(1/w_i^2 + 1/w_r^2)] \times (\sin 2ka - 2ka \cos 2ka) \sin \phi_s, \]

(17)

where \( \phi_s = \phi_{\text{GSW}}(r_s, z_s) \). This approximation enables us to perform the integration analytically even for off-axis positions of the sphere center (i.e., \( r_s \neq 0 \)).

According to Eq. (17) the sign of the force is determined by the product of two terms. The first one, \( \sin \phi_s \), comes from the spatial intensity profile described by the interference term in Eq. (9), and it determines the positions of zero, minimum, and maximum optical forces along the optical axis. The second term, \( \sin 2ka - 2ka \cos 2ka \), may be set equal to \( G_a \) and depends on the ratio of the particle size and the trapping wavelength in the medium. Its plot is shown in Fig. 2. Extremes of function \( G_a \) can be found for \( a_{\text{min}}/\lambda = M/4, M = 1, 2, \ldots \). The zero value of \( G_a \) is obtained for the following values of the sphere radius: \( a_{\text{min}}/\lambda = 0.3576, 0.6148, 0.8677, 1.1194, \ldots \). If a sphere of this particular size is placed into the GSW, the change in the field energy does not depend on the sphere position in the GSW. Consequently, the axial optical force acting on the sphere is zero for all particle positions in the GSW, and obviously this particle cannot be trapped.

3. \( a = w_0, a \ll z_0 \)

Let us consider the slightly more general case of a sphere that is so large that the intensity changes in the lateral direction cannot be neglected over the integration volume. To obtain analytical results we assume that the beam widths \( w_i(z) \) and \( w_r(z) \) do not change with respect to the particle size (see Appendix B) and that the particle is situated on the beam axis. Improved results with respect to the previous case can be expected only in a narrow range of particle sizes. Following Eqs. (11) and (12) the energy change and the axial force can be expressed as
\[
\Delta W(0, z_s) = -\alpha \frac{n_2}{c} P \left[ a(1 + \rho^2) - \frac{w_i}{\sqrt{2}} \text{daw} \left( \sqrt{2}a \frac{w_i}{w_r} \right) \right] + \rho^2 \frac{w_r}{\sqrt{2}} \text{daw} \left( \sqrt{2}a \frac{w_i}{w_r} \right) + \rho \frac{w_i}{w_0 w_r} \frac{2W^2}{k(1 + Z_w^2)} \sin(2ka)(\cos \phi_s - Z_w \sin \phi_s) + \text{Re} \left[ \frac{2W^3}{C^{3/2}} \exp(i\phi_s) \{ \exp(i2ka) \text{daw}(X + Y) + \exp(-i2ka) \text{daw}(-X + Y) \} \right], \tag{18}
\]

\[
F_z(0, z_s) = 4\alpha \frac{n_2}{c} P \frac{\rho}{w_0 w_r} \left( -\frac{W^2}{1 + Z_w^2} \sin(2ka) \right) \times (\sin \phi_s + Z_w \cos \phi_s) + \text{Re} \left[ \frac{i k W^3}{C^{3/2}} \exp(i\phi_s) \{ \exp(i2ka) \text{daw}(X + Y) + \exp(-i2ka) \text{daw}(-X + Y) \} \right], \tag{19}
\]

where \(X, Y, W, C, Z_w\), and \(\phi_s\) are defined in Appendix B.

3. GENERALIZED LORENZ–MIE THEORY APPLIED ON A SPHERE PLACED IN THE GAUSSIAN STANDING WAVE AND THE SINGLE BEAM

A. Calculation of Optical Forces by Using Generalized Lorenz–Mie Theory

The GLMT uses the scattering procedure presented first by Mie\textsuperscript{24} who derived expressions for the field distribution outside a spherical object of arbitrary size placed into a plane wave. This original method was generalized so that it permits a mathematical description of the forces acting on spherical and oblate objects placed in a general electromagnetic field.\textsuperscript{25,27,31} For the description of a focused beam, a modification to the fundamental Gaussian beam was introduced [so called fifth-order corrected Gaussian beam (CGB)].\textsuperscript{26,32} It uses a field expansion in the beam size parameter \(s = 1/kw_0\) to the fifth order, achieves better agreement with the wave equation, and therefore provides more precise calculation of the optical forces. Because we consider the standing wave created by the interference of two counterpropagating focused laser beams, we have easily adapted the GLMT formalism to this case. Instead of a single CGB we summed field components of two counterpropagating CGBs with overlapped beam waists to get the initial field components of the standing wave. Moreover, we assumed that the spherical object was located on the beam axis so we could employ the radial symmetry of the problem and simplify the calculation.\textsuperscript{33} We wrote the modified code ourselves, but we do not present a detailed mathematical description, because the method is well described in the literature.\textsuperscript{26,27,31,33}

In this study we neglected any electrostatic interactions between the surface and the particle as well as multiple scattering of the incident beam. Furthermore, we assumed that the beam waist was placed on the surface with reflectivity equal to 100\%. The axial positions \(z_s\) of the sphere center, where we calculated the axial forces, satisfy the inequality \(a \leq z_s \leq (a + \lambda)\).

Although the adopted simplifications (CGB, absence of spherical aberrations, diffraction, and multiple-scattering events) may seem drastic, this model provides at least a correct qualitative description of the behavior of dielectric spheres in the GSW\textsuperscript{34} and an acceptable speed of calculation.

B. Validity of the Electrostatic Approximation

Validity of the ESA is limited mainly by the assumptions that the object does not change the field distribution and that the incident beam is polarized only in the transverse direction [see Eq. (4)]. Therefore, it is desirable to study the differences between the ESA and a more precise method (GLMT) as a function of object size, refractive index, and Gaussian beam waist size.

Rough comparison of the GLMT and the ESA methods revealed that the ESA methods are applicable only if the object is smaller than 0.25\(a\). For larger spheres, only very small relative refractive indices must be used. Since the maximum trapping force is nonzero for particles of applicable sizes, we can compare the methods by using maximum relative error of the ESA trapping force defined by the formula

\[
\text{Fig. 4. Maximum relative error of the ESA methods (in percent) as a function of the sphere radius and relative refractive index for four beam waist sizes (}w_0=0.75, 1, 1.25, 2.5\lambda)\text{ and the following parameters: } \rho = 1, \psi = 3\pi/2, \lambda_{\text{vac}} = 1064 \text{ nm}, \ z_0 = 0 \mu\text{m.}
\]
where \( F_{\text{max}} = \max(F_{\text{GLMT}}(a < z_s < a + \lambda)) \) and \( F_{\text{apr}} \) is the force value calculated using a particular approximation “apr.” Relative errors are summarized in Fig. 4 for various beam waist sizes as a function of the sphere radius and the relative refractive index. The numbers on the contour curves mark the relative error in percent. We compared the GLMT with three ESA methods labeled ESA-int, ESA-daw, and ESA-anal calculated by using Eqs. (12), (B8), and (17), respectively. ESA-int is the most precise method, giving \( 5\% < \Delta F_z < 22\% \) for \( w_0 = 0.75\lambda \) and \( 0.5\% < \Delta F_z < 14\% \) for \( w_0 = 2.5\lambda \). The ESA-daw method gives more precise results than the ESA-anal one for intermediate sphere radii \( (0.1 < a/\lambda < 0.23) \) and smaller \( m \), but negligible improvement is found for greater \( m \). The relative error of the ESA-daw method satisfies \( 10\% < \Delta F_z < 34\% \) for \( w_0 = 0.75\lambda \) and \( 0.5\% < \Delta F_z < 16\% \) for \( w_0 = 2.5\lambda \). We see that the wider the beam waist size, the better the coincidence with the GLMT, but no significant improvement was found for \( w_0 > 2.5\lambda \).

We also checked whether intuitive substitution of \( \alpha \) by \( 3(\alpha^2 - 1)/(\alpha^2 + 2) \) in Eqs. (12), (B8), and (17), according to Eq. (15), improves the coincidence between the GLMT and the ESA methods. We denote these methods ESAn-int, ESAn-daw, and ESAn-anal, and results are shown in Fig. 5. We can see that precision increased on average by more than three times, especially for higher refractive indices and intermediate particles \( (0.1\lambda < \alpha < 0.22\lambda) \). The relative error is smaller than 5% for the majority of ESAn-int results. Contrary to the case with ESA-daw, the ESAn-daw method slightly increases the precision in a narrow region of higher \( m \) for small \( w_0 \) and smaller \( m \) for wider \( w_0 \). Together with the ESAn-anal method the relative error with the ESAn-daw method is smaller than 10% for wider beam waist sizes. No simple tendency with respect to the beam waist size was found, but there is a region of minimum error that moves from the area of higher \( m \) for smaller \( w_0 \) toward the area of smaller \( m \) for higher \( w_0 \). The steep increase of \( \Delta F_z \) for larger \( \alpha \) and \( m \) in Figs. 4 and 5 is caused by the fact that \( F_{\text{max}} \), which appears in the denominator of Eq. (20), decreases here.

C. Comparative Study of the Maximum Trapping Force in the Standing-Wave Trap and the Single-Beam Trap

Figure 6 presents the contour plots of the maximum axial trapping force acting on the spherical particles placed
into the GSW for various values of particle size, relative refractive index, and beam waist size. The numbers represent the levels of constant forces in piconewtons, and the shaded regions correspond to the negative maximum trapping force. For these parametric configurations the particle is accelerated toward the surface without a chance of its confinement in the GSW (nontrapping regions). This is caused by the gradient forces from the neighboring GSW antinodes, which pull the object in opposite directions and thus cancel each other. Consequently, the weaker gradient force due to the focused beam envelope dominates and accelerates the particle towards the beam waist placed on the mirror. The larger the sphere, the wider is this nontrapping region. We also see that this region becomes narrower with increasing beam waist since the gradient of the Gaussian envelope of the intensity becomes smaller.

Nontrapping predictions of the ESA models fit the GLMT results only for $m = 1$. Figure 6 shows that the difference between the consequent force-maximizing or -minimizing sphere radii is smaller than ESA predictions (equal to $\lambda/4$) and still decreases with increasing value of the relative refractive index.

Figure 7 shows the maximum trapping force in the SBT for comparison. We see that the wider the beam waist, the lower must be the relative refractive indices used to confine the object in the SBT. Direct comparison with the SWT forces (Fig. 6) acting on larger particles (outside the nontrapping regions) reveals that the SWT provides trapping forces that are at least one order of magnitude stronger. For smaller particles and wider beam waists this disproportion becomes even more pronounced.

4. SUMMARY AND CONCLUSIONS

In the work described in this paper the trapping properties of the Gaussian standing wave (GSW) were studied theoretically by using electrostatic approximation (ESA) and generalized Lorenz–Mie theory (GLMT). On the basis of the ESA, analytical formulas for optical forces acting on dielectric particles with relative refractive indices close to unity were derived. The validity of the ESA and its analytical formulas was tested by comparison with the numerical results obtained from the GLMT. Despite the drastic simplification, it turned out that the ESA methods are convenient for fast estimation of axial forces acting on particles of radii $a < 0.25\lambda$ and relative refractive indices $m < 1.21$ placed into the GSW of beam waist $w_0 > 0.75\lambda$. If the term $a = (m^2 - 1)$ is replaced by the term $3(m^2 - 1)/(m^2 + 2)$ taken from the Rayleigh scattering theory, the relative error of the ESA method with respect to the GLMT decreases below 16%.

The ESA model straightforwardly explains why spheres of radius smaller than $0.3\lambda$ are trapped in the antinodes and spheres of radius larger than $0.3\lambda$, but smaller than $0.55\lambda$ are confined in the nodes.

We found that the SWT, in contrast to the SBT, permits confinement of particles of high refractive indices even in moderately focused beams and could therefore be employed for easier optical confinement of such particles in air. This conclusion, however, is valid only for particle sizes and refractive indices outside the nontrapping regions, where the effect of the GSW is not minimized.

APPENDIX A: TIME-AVERAGED CHANGE IN FIELD ENERGY

If object and immersion media are linear (i.e., $D = \varepsilon E$, $B = \mu H$) and both media are nonmagnetic ($\mu = \mu_0$), Eq. (1) can be rewritten in the form

$$\Delta W = \frac{1}{2} \int_V (\mathbf{E}\mathbf{D}_0 - \mathbf{D}\mathbf{E}_0) dV$$

$$+ \frac{1}{2} \int_V (\mathbf{E} + \mathbf{E}_0)(\mathbf{D} - \mathbf{D}_0) dV. \quad (A1)$$
We get for the first integral in Eq. (A1) \( \int_V (E + E_0)(D - D_0) dV \)

\[
\frac{1}{2} \int_V \left[ - \nabla \phi_0 - \frac{\partial A_0}{\partial t} - \nabla \phi - \frac{\partial A}{\partial t} \right] (D - D_0) dV,
\]

\[
= \frac{1}{2} \int_V \left[ - \nabla (\phi_0 + \phi) - \frac{\partial (A_0 + A)}{\partial t} \right] (D - D_0) dV,
\]

\[
= \frac{1}{2} \int_V \nabla \phi_T (D - D_0) dV - \frac{1}{2} \int_V \frac{\partial A_T}{\partial t} (D - D_0) dV,
\]

(A2)

where new total potentials \( \phi_T = \phi_0 + \phi \) and \( A_T = A_0 + A \) were defined.

Since we assume harmonic fields, the second term in Eq. (A2) can be rewritten as

\[
- \frac{1}{2} \int_V \frac{\partial A_T (\sin \omega t)}{\partial t} (D - D_0) dV dV
\]

\[
= - \frac{1}{2} \int_V \omega A_T (D - D_0) \sin \omega t \cos \sin \omega t dV. \quad (A3)
\]

Therefore the time-averaged value over the field period is equal to zero:

\[
- \frac{1}{2} \left\langle \int_V \frac{\partial A_T}{\partial t} (D - D_0) dV \right\rangle_T = 0. \quad (A4)
\]

The first term in Eq. (A2) can be rewritten with use of integration by parts, as

\[
- \frac{1}{2} \int_V \nabla \phi_T (D - D_0) dV = - \frac{1}{2} \int_V \nabla \left[ \phi_T (D - D_0) \right] dV
\]

\[
+ \frac{1}{2} \int_V \phi_T \nabla (D - D_0) dV.
\]

(A5)

The second integral in Eq. (A5) is equal to zero if we make use of the fact that the sources of the field are unchanged during the placement of the dielectric particles, i.e., \( \nabla D = \nabla D_0 = \rho \), where \( \rho \) is the free-charge density. With the Gauss theorem, the first integral in Eq. (A5) can be rewritten as the integration of the normal component of the product \( \phi_T (D - D_0) \) over a surface embedding the object:

\[
- \frac{1}{2} \int_V \nabla \left[ \phi_T (D - D_0) \right] dV = - \frac{1}{2} \int_S \phi_T (D - D_0) \mathbf{n} dS.
\]

(A6)

If the radius \( R \) of the surface goes to infinity, the area increases as \( R^2 \) but the product \( \phi_T (D - D_0) \) for electrostatic fields decreases as \( R^{-3} \). However, this relationship is not valid for the radiation fields. To eliminate this integral, we apply Wolf's arguments even though they cannot be regarded as the rigorous ones. We assume that the radiation field does not exist at all times but instead is produced by some source at time \( t_0 \). At any time \( t > t_0 \), the field fills the region of space reaching to \( c(t - t_0) \) from the source (\( c \) is the velocity of the light). If the integration surface is far enough from the source, the field does not reach it within \( t - t_0 \), and the integral can be omitted. Therefore we can conclude that the time-averaged energy change caused by the insertion of the dielectric object into the electric field is described by

\[
\langle \Delta W_E \rangle_T = - \frac{1}{2} \int_V (\varepsilon_2 - \varepsilon_1)(E E_0)T dV. \quad (A7)
\]

Because of the time averaging, the equation takes the same form as in the electrostatic case.

**APPENDIX B: INTEGRATION IN THE GAUSSIAN STANDING WAVE IN SPHERICAL COORDINATES**

1. \( a \ll w_0 \)

Let us express the principal terms in Eq. (9) in spherical coordinates and simplify them as

\[
2 \frac{r_B^2}{w_{ir}^2} = 2 \left\{ \frac{r_s^2}{w_{ir}^2} + \frac{r_s^2 \sin \chi \cos \phi}{w_{ir}^2} + \frac{r_s^2 \sin^2 \chi}{w_{ir}^2} \right\}
\]

\[
= 2 \frac{r_s^2}{w_{ir}^2}, \quad (B1)
\]

\[
w_{sr}^2 = w_0^2 \left[ 1 + \left( \frac{z_s \pm z_0 + r \cos \chi}{z_R} \right)^2 \right], \quad (B2)
\]

\[
k \frac{1}{2} \frac{1}{R_{ir}} = \frac{k}{2} \frac{z_s \pm z_0 + r \cos \chi}{z_R} \left[ 1 + (z_s \pm z_0 + r \cos \xi)^2/z_R^2 \right]
\]

\[
= \frac{z_s \pm z_0}{z_R w_{ir}^2}, \quad (B3)
\]

\[
k \frac{r_B^2}{2} \left( \frac{1}{R_i} - \frac{1}{R_r} \right) = -r_s^2 \left[ \frac{z_s \pm z_0}{z_R w_{ir}^2} + \frac{z_s - z_0}{z_R w_{ir}^2} \right]. \quad (B4)
\]

Integration of noninterference terms in Eqs. (11) and (12) is equal to zero, or these terms can be taken in front of the integral. The rewritten interference term, with the substitution \( \xi = \cos \chi \) and \( \phi_s = \phi(r_a, z_s) \), gives for the force
Equation (16) can be obtained in similar manner.

2. \( a \ll z_R, a = w_0 \)

If the sphere is placed on axis \((r_s = 0)\) and is so large that the lateral variation of the intensity cannot be neglected, but the condition \( a \ll z_R \) is satisfied, the approximations in Eqs. (B2) and (B3) can be used again, but in stead of relations (B1) and (B4) the following expressions have to be employed:

\[
F_z(0, z_s) = \frac{a n_2}{c} I_0 \omega^2 \pi \alpha^2 \frac{w_0^2}{w_i w_r} \int_{-1}^{1} \exp[-a^2(1 - \xi^2)/W^2] \times \cos \left[ \phi_s + a \frac{\alpha}{R_i} \xi \right] \xi \, d\xi,
\]

\[
= \frac{a n_2}{c} I_0 \omega^2 \pi \alpha^2 \frac{w_0^2}{w_i w_r} \Re \left[ \exp(i \phi_s) \right] \times \int_{-1}^{1} \exp[-a^2 C(1 - \xi^2)/W^2] + 2ia k \xi \xi \, d\xi \],
\]

\[
= \frac{a n_2}{c} I_0 \omega^2 \pi \alpha^2 \frac{w_0^2}{w_i w_r} \left( \frac{W^2}{1 + Z_w^2} \sin(2ak) \right) \times \left( \sin \phi_s + Z_w \cos \phi_s \right) + \Re \left[ \frac{k W \sqrt{\pi}}{2C^{3/2}} \exp(i \phi_s - X^2 - Y^2) \right] \times [ \text{erf}(iX + iY) + \text{erf}(-iX + iY) ] \right),
\]

where \( \text{erf} \) and \( \text{Re} \) denote the error function and the real part, respectively, and

\[
W = w_i(z_s)w_r(z_s)\left( w_i^2(z_s) + w_r^2(z_s) \right)^{1/2},
\]

\[
X = -kW/\sqrt{C},
\]

\[
Y = a\sqrt{C}/W,
\]

\[
Z_w = W^2/R_i^2,
\]

\[
1/R_i^2 = (z_s + z_0)/l(z_p w_i^2) + (z_s + z_0)/l(z_p w_r^2).
\]

If the error function is transformed into the Dawson integral by using

\[
\text{erf}(ix) = \frac{2i}{\sqrt{\pi}} \exp(x^2) \text{daw}(x),
\]

\[
\text{daw}(x) = \int_{0}^{x} \exp(t^2 - x^2) \, dt,
\]

Eq. (19) can be obtained. Moreover, the expression for the energy change needs integration by parts of the noninterference and the interference terms, and Eq. (18) can be obtained in this way.

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