Optical conveyor belt based on Bessel beams

Tomáš Čižmář a, Veneranda Garcéz-Chávez b, Kishan Dholakia b and Pavel Zemánek a

a Institute of Scientific Instruments, Academy of Sciences of the Czech Republic, Královořecká 147, 612 64 Brno, Czech Republic

b School of Physics and Astronomy, University of St. Andrews, North Haugh, Fife, KY16 9SS, Scotland

ABSTRACT

In this paper we present the standing wave created from two counter-propagating non-diffracting (Bessel) beams as a device for confinement and precise delivery of sub-micron sized particles. The particle position in direction of beam propagation is controled by changing the phase shift between these two beams. We succeeded in delivery of polystyrene particles of diameter 410 nm over a distance of 300 µm. At the same time we experimentaly confirmed the theoretical prediction how the optical forces acting on particles in this kind of field depend on the size of the objects.

Keywords: optical force, Mie particle, colloidal particle, interference optical trap, Bessel beam

1. INTRODUCTION

The transfer of momentum from photons to micro-objects, nano-objects and even atoms enables manipulation with these objects. During the past 20 years it led to many unique experiments in many branches of physics, biology, and engineering.1, 2 The most popular setup for 3D manipulation is based on a single focused laser beam so called optical tweezers.3 Via movement of the laser focus this tool enables manipulation with particles of sizes from tens of nanometers to tens of micrometers immersed in water. The distance of the transport is given by the field of view of the microscope in lateral and axial plane together with low optical aberrations. Especially spherical aberration due to the refractive index mismatch of oil-immersion objectives4 limits the vertical range of transport. Here we present a solution that uses standing wave created from two counter-propagating beams. Changes in the phase in one of the beams cause the spatial movement of the whole structure of standing wave nodes and antinodes. If a particle is confined near the antinode (or node), it follows the motion of the antinode (or node). Therefore, by precise control of the phase it is possible to deliver sub-micron objects. To elongate the distance of transport we used so called non-diffracting (Bessel) beams that do not change their intensity spatial profile over a region of their existence.5 At the same time the radius of the core of these beams can be very narrow (units of micrometers) and these beams exist over much longer distance (hundreds of micrometers) than the Gaussian beams of comparable beam waist. To increase the number of particles delivered simultaneously we used the self-healing property of Bessel beams. Behind the obstacle they can restore their original spatial field distribution. Therefore, the objects already confined in the standing wave do not influence (so strongly as in Gaussian beam) their confined neighbor.

2. GENERAL THEORY

The concept of general vector non-diffracting beams was published by Bouchal.6 He showed that the decomposition of this beam to angular spectrum of plane waves7 enables to describe the incident non-diffracting beam as:

$$E^{(i)}(r) = \frac{-i k}{2 \pi} \int_{0}^{2 \pi} d \varphi \int_{0}^{\pi/2} d \Theta \sin \Theta A(\varphi, \Theta) \frac{\delta(\Theta - \Theta_{0})}{|\sin \Theta_{0}|} e^{i k r}$$

where $k \equiv (k_{x}, k_{y}, k_{z}) = (-k \sin \Theta \cos \varphi, -k \sin \Theta \sin \varphi, k \cos \Theta)$ is a wave-vector of plane wave with azimuthal angle $\Theta$ with respect to the $z$ axis (propagation axis of Bessel beam) and polar angle $\varphi$, $A(\varphi, \Theta)$ is an arbitrary

Further author information: (Send correspondence to P.Z.)
P.Z.: E-mail: pavlik@isibrno.cz, Telephone: +420 541 514 202
vector complex function and $\Theta_0 < \pi/2$. By choosing proper $A(\varphi, \Theta)$ it is possible to obtain a variety of non-diffracting beams including optical vortices.\(^6\)

In our case we are interested in the situation when incident linearly polarised beam passes through an axicon. Let us assume for further simplicity that the beam incident on the axicon is a plane wave propagating along axis $z$ and linearly polarised along axis $x$

$$E(z, t) = E_0 e^{i(kz - \omega t)} e_x.$$  (2)

The analysis of its passage revealed that behind the axicon the amplitude $A(\varphi, \Theta)$ can be written as:

$$A(\varphi, \Theta_0) = \frac{i}{k} E_0 A_0 \{[\cos \Theta_0 + \sin^2 \varphi (1 - \cos \Theta_0)] e_x - [(1 - \cos \Theta_0) \sin \varphi \cos \varphi] e_y + \sin \Theta_0 \cos \varphi e_z\},$$  (3)

where $e_j (j = x, y, z)$ are unit vectors of Cartesian co-ordinate system, $A_0$ is the strength factor that depends on the axicon properties:

$$A_0 = \frac{\sin \tau}{\sin(\tau - \Theta_0) \sqrt{\cos \Theta_0}},$$  (4)

where $\Theta_0$ is the azimuthal angle of wave-vector of the plane wave behind the axicon (it is the same for all plane waves in the spectrum) and $\tau$ is one half of the axicon cone angle.

If Eqs. (3,4) are substituted to (1) and integral representations of Bessel functions are applied (similarly to\(^7\)) the following equation can be obtained for the non-diffracting beam behind the axicon:

$$E^{(i)}(r) = \frac{1}{2} E_0 \{[I_0 + \cos(2\varphi_r) I_2] e_x + \sin(2\varphi_r) I_2 e_y - [2 \cos \varphi_r I_1] e_z\},$$  (5)

where

$$I_0 = A_0 (1 + \cos \Theta_0) J_0(k_x \rho) e^{ik_z z},$$  (6)
$$I_1 = A_0 \sin \Theta_0 J_1(k_x \rho) e^{ik_z z},$$  (7)
$$I_2 = A_0 (1 - \cos \Theta_0) J_2(k_x \rho) e^{ik_z z},$$  (8)
$$\rho = r \sin \theta_r \quad z = r \cos \theta_r,$$  (9)
$$k_x = k \sin \Theta_0 \quad k_z = k \cos \Theta_0,$$  (10)

where $r, \varphi_r, \theta_r$ are spherical co-ordinates of the position vector $r$ where the field is calculated.

Using similar procedure amplitude $A(\varphi, \Theta_0)$ for magnetic field can be found\(^7\):

$$A(\varphi, \Theta_0) = - \frac{i}{2k} E_0 A_0 \{-[(1 - \cos \Theta_0) \sin \varphi \cos \varphi] e_x + [1 - \sin^2 \varphi (1 - \cos \Theta_0)] e_y + \sin \Theta_0 \sin \varphi e_z\}$$  (11)

and it gives the following magnetic field component:

$$B^{(i)}(r) = \frac{k}{2\omega} E_0 \{[2 \cos \varphi_r I_2] e_x + [I_0 - \cos(2\varphi_r) I_2] e_y - [2i \sin \varphi_r I_1] e_z\}.$$  (12)

### 2.1. Standing Bessel beam

In the case of two identical counter-propagating Bessel beams the following equations can be found for the electric and magnetic field components:

$$E^{(SW)}(r) = E_0 \{[I_0^{SW} + \cos(2\varphi_r) I_2^{SW}] \cos(k_z z) e_x + \sin(2\varphi_r) I_2^{SW} \cos(k_z z) e_y + [2 \cos \varphi_r I_1^{SW}] \sin(k_z z) e_z\},$$  (13)

$$B^{(SW)}(r) = \frac{i k}{\omega} E_0 \{[\sin(2\varphi_r) I_2^{SW}] \sin(k_z z) e_x + [I_0^{SW} - \cos(2\varphi_r) I_2^{SW}] \sin(k_z z) e_y - [2i \sin \varphi_r I_1^{SW}] \cos(k_z z) e_z\},$$  (14)

where

$$I_0^{SW} = A_0 (1 + \cos \Theta_0) J_0(k_x \rho),$$  (15)
$$I_1^{SW} = A_0 \sin \Theta_0 J_1(k_x \rho),$$  (16)
$$I_2^{SW} = A_0 (1 - \cos \Theta_0) J_2(k_x \rho).$$  (17)
3. EXPERIMENTAL SETUP

The easiest way how to obtain well-aligned counter-propagating Bessel beams is to use reflection of the incident Bessel beam on the mirror. Unfortunately, in these cases the standing wave is observed along the direction of beams propagation and so the simultaneous 3D confinement of several objects is not clearly visible.

![Experimental setup diagram](image)

**Figure 1.** Experimental setup. Linearly polarized beam (IPG, 10 W, \( \lambda_{ac} = 1.070 \ \mu m \)) passed through half-wave plate \( \lambda/2 \) and was divided by a polarization beam-splitter PBS1 into two paths with controllable power ratio. The first p-polarized beam is reflected on metallic mirror M1, transformed to the Bessel beam by an axicon A1 (Eksma 130-0270). The core of this Bessel beam is decreased by a telescope T1 formed by lens L1 (\( f=38 \ \text{mm} \)) and microscope objective O1 (Newport, M-40X, N.A. 0.40). The second s-polarized beam is reflected on the metallic mirror M2 and on a polarization beam-splitter PBS2, passed through the quarter-wave plate \( \lambda/4 \) to the axially movable mirror M3 controlled by stepmotor (Newport ESP 300), reflected back through the same \( \lambda/4 \) (the beam became p-polarized) and transformed to the Bessel beam by an axicon A2 (Eksma 130-0260) and narrowed down by T2 made from lens L2 (\( f=38 \ \text{mm} \)) and microscope objective O2 (Newport, M-20x, N.A. 0.40). Both Bessel beams interfere in cuvette C. The particles are imaged by objective with long working distance (LDO) on a CCD camera using the scattered light.

Therefore, in this paper we present a different arrangement where the standing Bessel beam is created from two independent counter-propagating Bessel beams with alterable phase shift between them (see Fig. 1). Whilst slightly different optical components were used in both paths at the time of this experiment, the estimated widths and propagation distances of both counter-propagating beams are very close to each other. From the parameters of the optical system the diameters of the central Bessel beam cores are equal to 2.24 \( \mu m \) and 2.14 \( \mu m \) for the first or the second path, respectively. The cuvette was 5 mm long made from 1 mm thick BK7 glass and it was filled up with \( D_2O \). It has lower absorption at the trapping wavelength and so the heating and unwanted convection of the liquid was suppressed. The microparticles were put into the cuvette and the light scattered perpendicularly to the propagation of the beams was observed by a microscope objective LDO with long working distance and CCD camera.

The movement of the mirror M3 causes the phase shift between both counter-propagating beams in a controlled fashion and this resulted in the movement of the whole standing wave structure together with the confined sub-micron particles - a sort of optical micro-conveyor belt. Since the positioning accuracy of the stepmotor is 1 \( \mu m \), we could precisely deliver number of confined particles over a distance much longer than it was possible in Gaussian beam of width comparable to the core of the Bessel beam.

4. EXPERIMENTAL RESULTS

To the best of our knowledge this is the first demonstration of controlled 3D delivery of sub-micron particles using standing Bessel beam and at the same time the first example of 3D optical confinement of the smallest
particles using non-diffracting beams. We also note that we can induce a continuous phase shift between the two beams using the angular Doppler effect for continuous delivery of microparticles.\textsuperscript{9}

We succeeded in 3D optical trapping and delivery of polystyrene beads of diameters 410, 490, 600, 800, 930, and 1000 nm. Figure 2 shows examples of time evolution of single particle, two particles and a group of particles of 410 nm in diameter. Notably from these selection of particles sizes, those of diameter \(d=490\) and 800nm jumped much more easily between neighboring longitudinal optical traps and stayed confined in one trap for significantly shorter timescales. This behavior coincides with theoretical predictions based on the Generalized Lorenz-Mie scattering theory applied on the calculation of the optical forces in counter-propagating Bessel beam. The detailed description of this method will be the subject on future article but results related to this present experiment are shown in Fig. 3.

We also note from this figure that there are bead sizes that may not even be trapped in this interferometric scheme. In turn this opens up the prospect of selectively trapping and transporting sub-micron objects of given sizes and a new method of optically sorting them based upon their affinity to this periodic light pattern. This discrimination between particles of different size is a key feature of the standing wave geometry.\textsuperscript{10}

In order to compare theory with experimental results more precisely, we applied correlation algorithm\textsuperscript{11} to track the particles trajectory along both the x and z axis from the video records. Examples of histograms for particles of diameter 410 nm or 490 nm, respectively, is shown in Fig. 4. These results support the statement that smaller sphere is strongly confined since it stayed in one trap. On the other hand plenty of jumps between neighboring longitudinal traps occurred for bigger sphere and the distances between them fit very well to the distance between two neighboring standing wave antinodes \(\delta z = 419\text{nm}\). Unfortunately this experimental system was not stable enough and the small drift of the longitudinal traps smeared the clear distinction between neighboring traps during repeated particle returns to the same trap.

We calculated the trap stiffness from the time record of x and z bead positions. We choose ranges of the same number of consecutive positions (50), eliminated the drift by fitting a line and calculated root-mean-square
Figure 3. The theoretical dependence of the extremal (maximal or minimal) axial optical force (left plot) and trap depths $dU_z$ (right plot) in the Bessel standing wave as a function of polystyrene beads diameter. Particles were confined in $D_2O$ using on-axis optical intensity $I_0 = 0.637 \, W \, \mu \text{m}^{-2}$ in each beam (equivalent to on-axis intensity in Gaussian beam of beam waist $1 \, \mu \text{m}$ and power $1 \, \text{W}$) of the idealized Bessel beam created by the same optics as in the experiment. The negative values of extremal forces indicates that the bead is trapped with its centre at node of the SW. Stars denote the beads used in experiments. Vertical dotted lines indicate bead sizes that are not affected by the Bessel standing wave and therefore cannot be confined in this periodic structure.

deviations (RMSD)$^{12} < x^2 >$ and $< z^2 >$ from this line. The trap stiffness is inversely proportional to the RMSD. Unfortunately we could not compare the experimental stiffness with theoretical predictions because we did not know the exact trapping laser intensity at the place of particle confinement. This quantity is not experimentally available since the ideal Bessel beam is in reality longitudinally modulated by Gaussian envelope and so it is dependent on the axial position of the bead. For this reason during the exchange of the bead sizes we tried to put the cuvette to the same places and kept the same laser power. Nevertheless we could compare the ratio of stiffnesses $\frac{\kappa_z}{\kappa_x} = \frac{< x^2 >}{< z^2 >}$, that is insensitive to trapping power (see Fig. 5).

We believe that this comparison reflects the theoretical trends even though the error bars indicate that much more attention must be paid to the stability of the system in future measurements. The ratio is bigger than 2 for the majority of studied particle diameters and therefore the objects are confined tighter longitudinally than laterally. In contrast for a single focused Gaussian beam where this ratio lies between 0.68 for $d=400\, \text{nm}$ and
Figure 4. Histogram of $x$ and $z$ positions of polystyrene spheres confined in Bessel standing wave. The smaller sphere is much better confined near one optical trap over a period 166 s. The bigger one jumps several times between neighboring traps even though the time period was shorter and equal to 127 s. The other parameters of the system like laser power, beam setup and polarization were the same.

Figure 5. Comparison of the ratio of longitudinal and lateral stiffnesses $\kappa_z / \kappa_x$ obtained from the theory (the same parameters as in Fig. 3) and experiment. The error bars correspond to the 90% confidence level.

0.2 for $d=1000\text{nm}$, respectively. For any given size of trapped bead, if a Gaussian beam is simply focused to a diffraction limited spot, there is a fixed relation between the maximum trapping force and the stiffness of the trap. However, the elasticity of biological molecules is very non-linear and to get higher grade information, there is a need to locate the molecules more precisely in one dimension than another and an asymmetry in stiffness in the optical trap as demonstrated here might facilitate such studies.

5. CONCLUSIONS

Using the Bessel beam for 3D confinement of micro-particles in standing wave enables to generate a much longer array of optical traps comparing to the Gaussian beam. Therefore this choice of beam is very useful method for manipulation with micro-objects even though the power requirement is large. We have demonstrated how two counter-propagating Bessel beams can be used to create standing wave where the sub-micron particles can be 3D confined and precisely delivered over a distance of hundreds of $\mu\text{m}$ by sliding the Bessel standing wave. This optical conveyor belt may have potential for the delivery of biological and colloidal microparticles and indeed a variant of this system may also be used for deterministic delivery of cold atoms.
6. ACKNOWLEDGMENTS

This work was partially supported by the Institutional Research Plan of the Institute of Scientific Instruments (AV0Z20650511), Grant Agency of the Academy of Sciences (IAA1005203), European Commission via 6FP NEST ADVENTURE Activity (ATOM3D, project No. 508952), and European Science Foundation (02-PE-SONS-063-NOMSAN project).

REFERENCES